Mathematics Level C: Lesson 1 – Equivalent Fractions

Targeted Standards:
4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Suggested Lesson Structure
- Fluency Practice (12 minutes)
- Application Problem (8 minutes)
- Concept Lesson (30 minutes)
- Student Debrief (10 minutes)
- Total Time (60 minutes)

Fluency Practice (12 minutes)
- Sprint: Equivalent Fractions Fluency Practice
  [Note: A “Sprint” is a timed practice of a specific skill, usually on a single worksheet that is repeated so that students can improve their time and accuracy.]

Equivalent Fractions (3 minutes)
T: (Write.) \( \frac{1}{2} = \)
T: Say the fraction.
S: One half.
T: (Write.) \( \frac{1}{2} = \frac{1}{4} \)
T: One half is how many fourths?
S: Two fourths.

Continue with possible sequence:
\[
\begin{align*}
\frac{1}{2} & = \frac{1}{6} = \frac{2}{12} = \frac{2}{6} = \frac{2}{3} = \frac{3}{6} = \frac{3}{12} = \frac{3}{25}
\end{align*}
\]
T: (Write.) \( \frac{1}{2} = \)
T: Say the fraction.
S: One half.
T: (Write.) \( \frac{1}{2} = \frac{2}{6} \)
T: One half or one part of two is the same as two parts of what unit?
S: Fourth.

Repeat with another possible sequence:
Mr. Hopkins has a 1-meter wire he is using to make clocks. Each fourth meter is marked off with 5 smaller equal lengths. If Mr. Hopkins bends the wire at \( \frac{3}{4} \) meter, what fraction of the marks is that?

T: (After the students have solved the problem, possibly using the RDW process independently or in partners.) Let’s look at two of your solutions and compare them.

T: When you look at these two solutions side by side what do you see? (You might use the following set of questions to help students compare the solutions as a whole class, or to encourage inter-partner communication as you circulate while they compare.)

- What did each of these students draw?
- What conclusions can you make from their drawings?
- How did they record their solutions numerically?
- How does the tape diagram relate to the number line?
- What does the tape diagram/number line clarify?
- What does the equation clarify?
- How could the statement with the number line be rephrased to answer the question?
Concept Development (30 minutes)

Materials: (S) Blank paper

Problem 1

\[ \frac{1}{3} + \frac{1}{3} \]

1 third + 1 third = 2 thirds

T: On a number line, mark the end points as zero and 1. Between zero and 1 estimate to make three parts of equal length and label them with their fractional value.

T: (After students work.) On your number line, show 1 third plus 1 third with arrows designating lengths. (Demonstrate and then pause as students work).

T: The answer is?
S: 2 thirds.
T: Talk to your partner. Express this as a multiplication equation and as an addition sentence.

S: \[ \frac{1}{3} + \frac{1}{3} = 2 \times \frac{1}{3} = \frac{2}{3} \]

T: Following the same pattern of adding unit fractions by joining lengths, show 3 fourths on a number line.

Problem 2

\[ \frac{3}{8} + \frac{3}{8} + \frac{1}{8} \]

3 eighths + 3 eighths + 1 eighth

T: On a number line, again mark the end points as zero and one. Between zero and one, estimate to make 8 parts of equal length. This time only label what is necessary to show 3 eighths.

T: (After students work.) Represent 3 eighths + 3 eighths + 1 eighth on your number line.

T: The answer is?
S: 7 eighths.
T: Talk to your partner. Express this as a multiplication equation and as an addition equation.

S: \[ \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = (2 \times \frac{3}{8}) + \frac{1}{8} = \frac{7}{8} \]
Lesson 2: Make equivalent fractions with sums of fractions with like denominators.

Problem 3

\[
\frac{6}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = 1 + 1 + 1 = 3
\]

6 halves = 3 x 2 halves = 3 ones = 3

T: On a number line, mark the end points as 0 halves and 6 halves below the number line. Estimate to make 6 parts of equal length. This time only label 2 halves.

T: (After students work.) Record the whole number equivalents above the line.

T: Represent 3 x 2 halves on your number line.

T: (After students have worked) The answer is?

S: 6 halves or 3.

T: 3. What is the unit?

S: 3 ones.

T: Talk to your partner. Express this as an addition equation and as a multiplication equation.

S: \[
\frac{6}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = 3 \times \frac{2}{2} = 3
\]

Problem 4

\[
\frac{8}{5} = \frac{5}{5} + \frac{3}{5} = 1 \frac{3}{5}
\]

8 fifths = 5 fifths + 3 fifths = 1 and 3 fifths

T: Use a number line. Mark the end points as 0 fifths and 10 fifths below it. Estimate and give a value to the halfway point.

T: What will be the value of the halfway point?

S: 5 fifths.

T: Make 10 parts of equal length from 0 fifths to 10 fifths.

T: (After students work.) Record the whole number equivalents above the line.

T: (After students work.) Label 8 fifths on your number line.

T: Show 8 fifths as the sum of 5 fifths and 3 fifths on your number line.

S: (After students work.)

T: Talk to your partner. Express this as an addition equation in two ways: as the sum of fifths and as the sum of a whole number and fifths.

T: (After students work.) Another way of expressing 1 plus 3 fifths is?

S: 1 and 3 fifths.

S: \[
\frac{6}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = 3 \times \frac{2}{2} = 3
\]

T: 8 fifths is between what 2 whole numbers?

S: 1 and 2.
Lesson 2: Make equivalent fractions with sums of fractions with like denominators.

Lesson Objectives:
- Functions and Context Standards: How do they work together in instruction

Problem 5

\[
\frac{7}{3} = \frac{6}{3} + \frac{1}{3} = 2 \times \frac{3}{3} + \frac{1}{3} = 2 + \frac{1}{3} = 2\frac{1}{3}
\]

7 thirds = 6 thirds + 1 third = 2 and 1 third.

T: Use a number line. Mark the end points as 0 thirds and 9 thirds below the number line. Divide the whole length into three equal smaller lengths and mark their values using thirds. Work with a partner.

T: (After students work.) What are the values of those points?

S: 3 thirds and 6 thirds.

T: Mark the whole number equivalents above the line.

T: (After students work.) Divide each of those whole number lengths into three smaller lengths. Mark the number 7 thirds.

T: (After students work.) Show 7 thirds as two units of 3 thirds and one more third on your number line and in an equation. Work together if you get stuck.

T: (After working and dialogue) 7 thirds is between what two whole numbers?

S: 2 and 3.

Student Debrief (10 minutes)

Lesson Objective: Making equivalent fractions with sums of fractions with like denominators.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

T: Come to the Debrief and bring your Problem Set. Compare your work to your neighbor’s. On which problems do you have different answers? Discuss your differences. Both may be correct.

T: (After about 3 minutes.)

T: What is a way to express 3/7 as a sum?

S: 1 sevenths + 1 seventh + 1 seventh.

T: Another way?

S: 2 sevenths + 1 seventh.

T: These are equivalent forms of 3 sevenths.

T: On your Problem Set find and talk to your partner about different equivalent forms of your numbers.

S: 6 sevenths could be expressed as 3 sevenths + 3 sevenths or as 3 times 2 sevenths. → 9 sevenths can be expressed as 1 + 2 sevenths. → 7 fourths can be expressed as 2 times 3 fourths + 1 fourth. → 1 and 3 fourths can be expressed as 7 fourths. → 32 sevenths can be expressed as 28 sevenths + 4 sevenths or 4 and 4 sevenths.
Lesson 2:

Make equivalent fractions with sums of fractions with like denominators.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

Student Sample:

3) Express each of the following as the sum of a whole number and a fraction. Show c and d on number lines.

   a) \( \frac{9}{7} \) = \( \frac{7}{7} \) + \( \frac{2}{7} \) = \( 1 + \frac{2}{7} \) = \( 1\frac{2}{7} \)

   b) \( \frac{9}{2} \) = \( 3 \times \frac{3}{2} \) + \( \frac{1}{2} \) = \( 3\frac{1}{2} \)

   c) \( \frac{32}{7} \) = \( \frac{28}{7} \) + \( \frac{4}{7} \) = \( 4\frac{4}{7} \)

   d) \( \frac{24}{9} \) = \( \frac{18}{9} \) + \( \frac{6}{9} \) = \( 2\frac{6}{9} \)

4) Marisela cut four equivalent lengths of ribbon. Each was 5 eighths of a yard long. How many yards of ribbon did she cut? Express your answer as the sum of a whole number and the remaining fractional units. Draw a number line to represent the problem.

   \( \frac{2}{8} \) + \( \frac{16}{8} \) + \( \frac{15}{8} \) + \( \frac{20}{8} \) = \( 2\frac{1}{8} \) yards long.